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# On Non-Invariant Hypersurface with para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure of P-Sasakian Manifold

Vivekanand yadav[1] and Dhruwa Narain[2]
[1,2]*Department of Mathematics and Statistics, D.D.U Gorakhpur University, Gorakhpur-273009*

***Abstract:*** - The object of present paper is to study the hypersurface $\tilde{M}$ of a P-Sasakian manifold $M$ equipped with para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure and investigated the results when $\tilde{\phi}, U, V$ are parallel fields. Some properties of normal para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure also have been studied.

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## I. INTRODUCTION

Let $M$ be an $n$-dimensional differentiable manifold on which there exists a $(1,1)$ tensor field $\phi$, a vector field $\xi$ and $1$ −from $\eta$ satisfying

(1.1) $\phi^2 = I - \eta \otimes \xi$

(1.2) $\eta(\xi) = 1$

(1.3) $\eta o \phi = 0$

(1.4) $\phi \xi = 0$

is called an almost para contact manifold and the structure $(\phi, \xi, \eta)$ is called an almost para contact structure.
Let $g$ be a Riemannian metric with$(\phi, \xi, \eta)$ -structure such that

(1.5) $g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$

or ,equivalently,

(1.6) $g(\phi X, Y) = g(X, \phi Y)$ and $g(X, \xi) = \eta(X)$

for all vector fields $X, Y$.

Then $M$ is called an almost para contact Riemannian manifold or an almost para contact metric manifold with an almost para contact Riemannian structure-$(\phi, \xi, \eta, g)$.

**Definition:** An almost para contact Riemannian manifold is called P-Sasakain manifold if

(1.7) $(\nabla_X \phi)(Y) = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi$

for all vector fields $X, Y$.

where $\nabla$ denotes the operator of co-variant differentiation with respect to Riemannian metric $g$.

On P-Sasakian manifold, we have

(1.8) $(a)$ $(\nabla_X \eta)(Y) = g(\phi X, Y) = (\nabla_Y \eta)(X)$

$(b)$ $(\nabla_X \eta)(Y) = \Phi(X, Y)$ where $\Phi(X, Y) \stackrel{\text{def}}{=} g(\phi X, Y)$

$(c)$ $(\nabla_X \xi) = \phi X$

## II. HYPERSURFACE OF A P-SASAKIAN MANIFOLD WITH PARA $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ − STRUCTURE

Let $M$ be $n$ dimensional Riemannian manifold. Let us consider a $(n-1)$ dimensional differentiable manifold $\tilde{M}$ embedded in $M$ with embedding $b : \tilde{M} \longrightarrow M$ . The map $b$ induces a linear transformation map (called Jacobi map) $B: T_p \longrightarrow T_{bp}$ .

Let an affine normal $N$ of $\tilde{M}$ is in such a way that $\phi N$ is always tangent to the hypersurface and satisfying the linear transformations.

(2.1) $\phi BX = B\tilde{\phi} X + u(X)N$

(2.2) $\phi N = BU$

(2.3) $\xi = BV + \lambda N$

(2.4) $\eta(BX) = v(X)$

where $\tilde{\phi}$ is $(1,1)$ type tensor ; $U, V$ vector fields ; $u, v$ are $1-$ form and $\lambda$ is a $c^{\infty}-$ function. If $u \neq 0$ , we call $\tilde{M}$ a non-invariant hypersurface of $M$ [2 ].

Operating $(2.1),(2.2),(2.3) and (2.4)$ by $\phi$ and using $(1.1)$, $(1.2),(1.3) and$ $(1.4)$ and taking tangent and normal parts separately, we get the following induced structure on $\tilde{M}$,

(2.5) $(a)$ $\tilde{\phi}^2 X = X - u(X)U - v(X)V$

$(b)$ $u(\tilde{\phi}X) = -\lambda v(X)$, $v(\tilde{\phi}X) = -\lambda u(X)$

$(c)$ $\tilde{\phi}U = -\lambda V$, $\tilde{\phi}V = -\lambda U$

$(d)$ $u(U) = 1-\lambda^2$, $u(V) = 0$

$(e)$ $v(U) = 0$, $v(V) = 1-\lambda,^2$ where $\eta(N) = \lambda$

From (1.5) and (1.6), we get the induced metric $\tilde{g}$ on $\tilde{M}.i.e$;

(2.6) $\tilde{g}(\tilde{\phi}X,\tilde{\phi}Y) = \tilde{g}(X,Y) - u(X)u(Y) - v(X)v(Y)$

(2.7) $\tilde{g}(U,X) = u(X)$, $\tilde{g}(V,X) = v(X)$

(2.8) $\tilde{\Phi}(X,Y) = \tilde{\Phi}(Y,X)$

where $\tilde{g}(\tilde{\phi}X,Y) \stackrel{\text{def}}{=} \tilde{\Phi}(X,Y)$

A manifold $\tilde{M}$ with a metric $\tilde{g}$ satisfying $(2.5),(2.6)$ *and* $(2.7)$ is called manifold with $(\tilde{\phi},\tilde{g},u,v,\lambda)$ −structure [ 3 ].Let $\tilde{\nabla}$ be the induced connection on the hypersurface $\tilde{M}$ of the affine connection $\nabla$ of $M$.

Now using Gauss and Weingarten`s equations

(2.9) $\nabla_{BX} BY = B\tilde{\nabla}_X Y + h(X,Y)N$

(2.10) $\nabla_{BX} N = -BHX + \omega(X)N$

where $h(X,Y) \stackrel{\text{def}}{=} \tilde{g}(HX,Y)$

Here $h$ and $H$ are second fundamental tensors of type (0,2) and (1,1) respectively and $\omega$ is $1-$ from.Now differentiating $(2.1),(2.2),(2.3) and (2.4)$ covariantly and using $(2.9),(2.10),(1.7)$ and reusing$(2.1),(2.2),(2.3) and (2.4)$, we get the following theorem;

**Theorem** $(\mathbf{2.1})$: Let $\tilde{M}$ be the hypersurface with $(\tilde{\phi},\tilde{g},u,v,\lambda)-$ structure of a SP-Sasakian manifold $M$, then we have

(2.11) $(a)$ $(\tilde{\nabla}_X\tilde{\phi})(Y) = -\tilde{g}(X,Y)V - v(Y)X + 2v(X)v(Y)V + u(Y)HX + h(X,Y)U$

$(b)$ $(\tilde{\nabla}_X u)(Y) = -\lambda\tilde{g}(X,Y) + 2\lambda v(X)v(Y) - u(Y)\omega(X) - h(X,\tilde{\phi}Y)$

$(c)$ $(\tilde{\nabla}_X v)(Y) = \tilde{g}(\tilde{\phi}X,Y) + \lambda h(X,Y)$

$(d)$ $\tilde{\nabla}_X V = \tilde{\phi}X + \lambda HX$

$(e)$ $\tilde{\nabla}_X U = \omega(X)U - \tilde{\phi}HX - \lambda X + 2\lambda v(X)V$

$(f)$ $h(X,U) = \lambda^2 v(X)$

$(g)$ $h(X,V) = u(X) - X\lambda - \lambda\omega(X)$

Since $\tilde{g}(HX,Y) = h(X,Y)$ then from (2.7) and(2.11)$(f)$, we have

**Cor.** $(\mathbf{2.1})$: $h(X,U) \neq 0$

$\Rightarrow HU = 0$

## III. PARALLEL VECTOR FIELDS WITH RESPECT TO INDUCED CONNECTION

Let $\tilde{M}$ be the non-invariant hypersurface with para $(\tilde{\phi},\tilde{g},u,v,\lambda)$ −structure of a P-Sasakian manifold $M$. A vector field $P$ is parallel with respect to the connection $\tilde{\nabla}$ if $\tilde{\nabla}_X P = 0\ \forall\ X \in \Gamma(\tilde{M})$.

**Theorem** $(\mathbf{3.1})$: If $\tilde{\phi}$ is parallel vector field with respect to induced connection in the non- invariant hypersurface $\tilde{M}$ with para $(\tilde{\phi},\tilde{g},u,v,\lambda)-$ structure of a P-Sasakian manifold $M$, then we have

(3.1) $(1-\lambda^2)h(X,Y) = u(X)v(Y) - \lambda^2 u(Y)v(X)$

(3.2) $h(X,V) = u(X)$

(3.3) $\omega = -d(log\lambda)$

**Proof:** If $\tilde{\phi}$ is parallel vector field, then $\tilde{\nabla}_X\tilde{\phi} = 0$ .Then from (2.11) $(a)$, we have

$$-\tilde{g}(X,Y)V - v(Y)X + 2v(X)v(Y)V + u(Y)HX + h(X,Y)U = 0$$

operating above by $u$ and using (2.5) and (2.11)$(f)$, we get (3.1).

Now putting $Y = V$ in (3.1) , we have (3.2).

Now from (2.11)$(g)$ and (3.2) we get

$$\omega = -d(log\lambda)$$

**Theorem $(3.2)$:** If $U$ is parallel vector field with respect to induced connection in the non- invariant hypersurface $\tilde{M}$ with para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure of a P-Sasakian manifold $M$, then we have

$(3.4)(a)$ $\quad \omega = -\frac{1}{2} d\{\log(1 - 2\lambda^2)\}$

$(b)$ $\quad h(X, \tilde{\phi}Y) = \lambda \tilde{g}(X, Y) - u(X)w(Y) - 2\lambda v(X)v(Y)$

**Proof:** Since $U$ is parallel vector field , we have $\tilde{\nabla}_X U = 0$

From $(2.11)(e)$, we get

(3.5) $\quad \omega(X)U - \tilde{\phi}HX - \lambda X + 2\lambda v(X)V = 0$

Operating (3.5) by $u$ , we get (3.4) $(a)$

Again from(3.5), we have

$\omega(X)u(Y) - \tilde{g}(\tilde{\phi}HX, Y) - \lambda\tilde{g}(X, Y) + 2\lambda v(X)v(Y) = 0$

Using $\tilde{g}(\tilde{\phi}X, Y) = \tilde{g}(X, \tilde{\phi}Y)$ and $\quad h(X, Y) \stackrel{\text{def}}{=} \tilde{g}(HX, Y)$, we get(3.4) $(b)$.

**Theorem $(3.3)$:** If $V$ is parallel vector field with respect to induced connection in the non- invariant hypersurface $\tilde{M}$ with para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure of a P-Sasakian manifold $M$, then we have

$(3.6)(a)$ $\quad \omega = -d(\log \lambda)$

$(b)$ $\quad (\tilde{\nabla}_X v)(Y) = 0$

**Proof:** Since $V$ is parallel vector field, we have $\tilde{\nabla}_X V = 0$

From $(2.11)(d)$, we get

(3.7) $\quad \tilde{\phi}X + \lambda HX = 0$

Operating (3.7) by $v$ we get (3.6) $(a)$

Further from(3.7) , we have

$$\tilde{g}(\tilde{\phi}X, Y) + \lambda\tilde{g}(HX, Y) = 0$$

Using$(2.11)(c)$, we have $\quad (\tilde{\nabla}_X v)(Y) = 0$

**Theorem $(3.4)$:** The Nijenhuis tensor of the non-invariant hypersurface $\tilde{M}$ with para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure of a P-Sasakian manifold $M$ is given by

(3.8) $$N(X, Y) = u(X)\{(\tilde{\phi}H - H\tilde{\phi})Y\} + u(Y)\{(H\tilde{\phi} - \tilde{\phi}H)X\} + \tilde{g}\{(H\tilde{\phi} - \tilde{\phi}H)X, Y\}U - 2\lambda\{(u \wedge v)(X, Y)\}V$$

**Proof:** The Nijenhuis tensor $N$ of $\tilde{\phi}$ is given by

(3.9) $\quad N(X, Y) = (\tilde{\nabla}_{\tilde{\phi}X}\tilde{\phi})(Y) - (\tilde{\nabla}_{\tilde{\phi}Y}\tilde{\phi})(X) + \tilde{\phi}(\tilde{\nabla}_Y\tilde{\phi})(X) - \tilde{\phi}(\tilde{\nabla}_X\tilde{\phi})(Y)$

Using equation (2.11) $(a)$ in equation (3.9), we get (3.8).

**Corollary $(3.1)$:** The Nijenhuis tensor of the non-invariant hypersurface $\tilde{M}$ with para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure of a P-Sasakian manifold $M$ vanishes if $\tilde{\phi}H = H\tilde{\phi}$ and $u \wedge v = 0$

**Proof:** Putting $\tilde{\phi}H = H\tilde{\phi}$ and $u \wedge v = 0$ in equation (3.8) we get $N(X, Y) = 0$

## 4 Normal para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure:

A para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure is said to be normal if the torsion tensor $S$ of $\tilde{\phi}$ satisfies

(4.1) $\quad S(X, Y) = N(X, Y) + du(X, Y)U + dv(X, Y)V = 0$

Where $N$ is Nijenhuis tensor and

(4.2) $\quad du(X, Y) = (\tilde{\nabla}_X\tilde{\phi})(Y) - (\tilde{\nabla}_Y\tilde{\phi})(X)$

(4.3) $\quad dv(X, Y) = (\tilde{\nabla}_X v)(Y) - (\tilde{\nabla}_Y v)(X)$

**Theorem $(4.1)$:** Let$\tilde{\phi}H = H\tilde{\phi}$ , then non-invariant hypersurface $\tilde{M}$ with para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure of P-Sasakian manifold $M$ is normal if, $u \wedge w = 0$ and $u \wedge v = 0$

**Proof:** Using equations (2.11) $(a)$, (2.11) $(b)$(2.11) $(c)$ in(4.2) *and* (4.3) we have

$$du(X, Y) = u(X)w(Y) - u(Yw(X) + \tilde{g}\{(H\tilde{\phi} - \tilde{\phi}H)X, Y\}$$

and $\quad dv(X, Y) = 0$

Using above two equations and (3.8) in(4.1) we get

$$S(X, Y) = u(X)\{(\tilde{\phi}H - H\tilde{\phi})Y\} + u(Y)\{(H\tilde{\phi} - \tilde{\phi}H)X\} + 2\tilde{g}\{(H\tilde{\phi} - \tilde{\phi}H)X, Y\}U + (u \wedge w)(X, Y)U - 2\lambda(u \wedge v)(X, Y)V$$

Now putting $\tilde{\phi}H = H\tilde{\phi}$, $u \wedge w = 0$ and $u \wedge v = 0$ , yields $S = 0$.

**Theorem $(4.2)$:** If the hypersurface $\tilde{M}$ with para $(\tilde{\phi}, \tilde{g}, u, v, \lambda)$ −structure of P-Sasakian manifold $M$ is normal then , we have

(4.4) $\eta^{\alpha}\big(N(X,Y)\big) + (1-\lambda^2)d\eta^{\alpha}(X,Y) = 0$
(4.5) $\overline{\overline{N(X,Y)}} - \lambda^2 N(X,Y) = 0$
Where $\bar{X} = \tilde{\phi}X$ and $\alpha = 1,2; \eta^1 = u, \eta^2 = v$

**Proof:** Operating $(4.1)$ by $\eta^{\alpha}$ we have $(4.4)$. Barring $(4.1)$ twice and using $(2.5)$ $(a)$ , we get
(4.6) $\overline{\overline{N(X,Y)}} + \lambda^2\{du(X,Y)U + dv(X,Y)V\} = 0$
Now multiplying $(4.1)$ by $\lambda^2$ and subtracting with $(4.6)$ ,we get $(4.5)$.

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